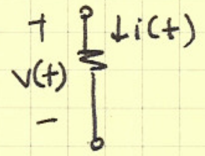
Non-linear resistors:

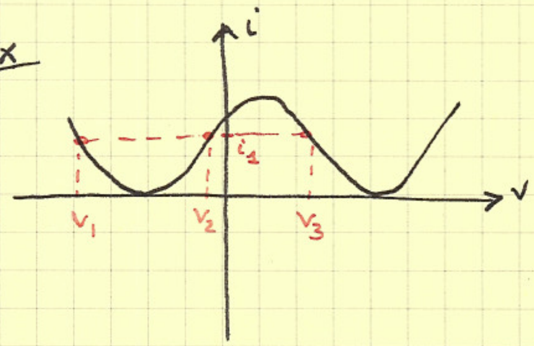


① Voltage controlled non-linear resistor

$$i(t) = f(v(t))$$

For each value of v, there is one and only one value of i (i.e. value of i is uniquely specified.)

ex



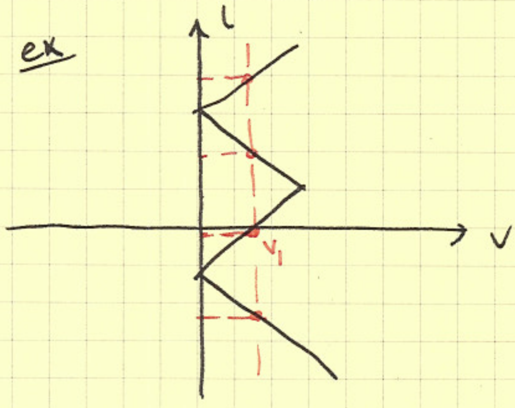
Draw lines perpendicular to v-axis. You should encounter a single i value along each line. (eg. pn-jnc diode, tunnel diode.

② Current controlled non-linear resistor

$$v(t) = g(i(t))$$

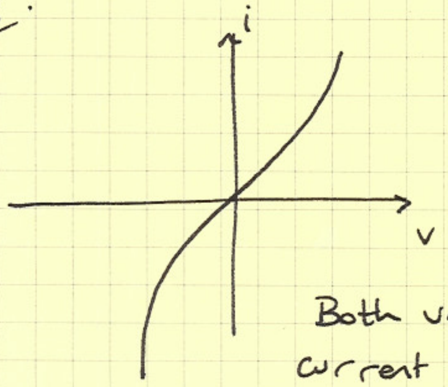
For each value of i, there is one and only one value of v. (eg. glow tube)

ex



Draw lines perpendicular to i-axis. You should encounter a single v-value along each line.

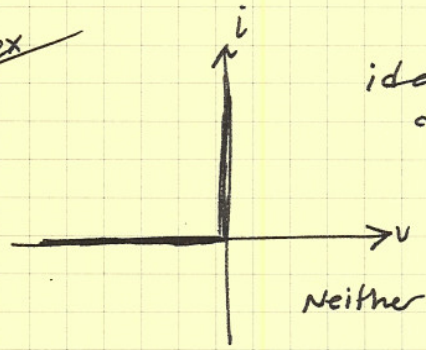
ex.



$i = v^3$

Both voltage and current controlled.

ex



ideal diode

Neither voltage nor current controlled.

Non linear Resistive Circuits:

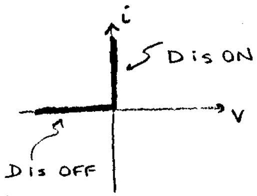
Diode: The diode is a two terminal, non-linear device that presents a relatively low resistance to current flow in one direction, and a relatively high resistance in the opposite direction.

Diode is NON-bilateral.

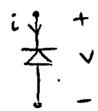
Ideal Diode: The ideal diode represents no resistance to current flow in the forward direction and an infinite resistance to current flow in the reverse direction.

D is ON: $v=0$ if $i\geq 0$

D is OFF: $i=0$ if $v\leq 0$

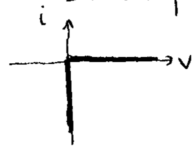


OR:

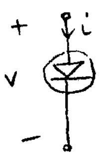


D is ON: $v=0$ if $i\leq 0$

D is OFF $i=0$ if $v\geq 0$



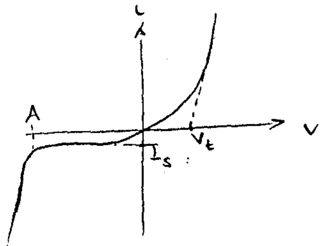
pn-junction Diode:



$$i = I_s\left(e^{v/v_t} - 1\right)$$

I_s: Reverse saturation current (current in the diode when it is reverse biased with a large voltage)

V_t: Threshold voltage (forward voltage required to reach the region of upward swing)

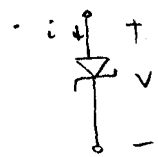


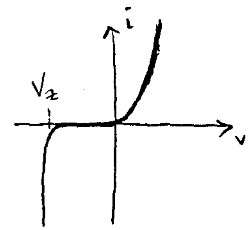
A: break down voltage (a few hundred volts)

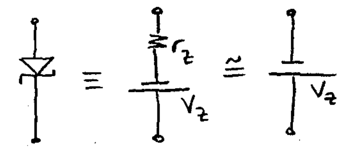
Normal operating range: Right of point A.

* pn junction diode is a voltage controlled non-linear resistor.

Zener Diode:

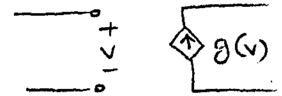






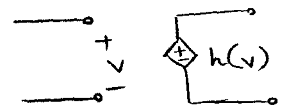
Non-linear Dependent Sources:

1) Voltage controlled current source: (VCCS)

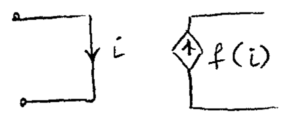


ex: $g(v) = \beta e^{v}$

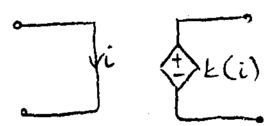
2) Voltage controlled voltage source (VCVS)



3) Current controlled current source (CCCS)



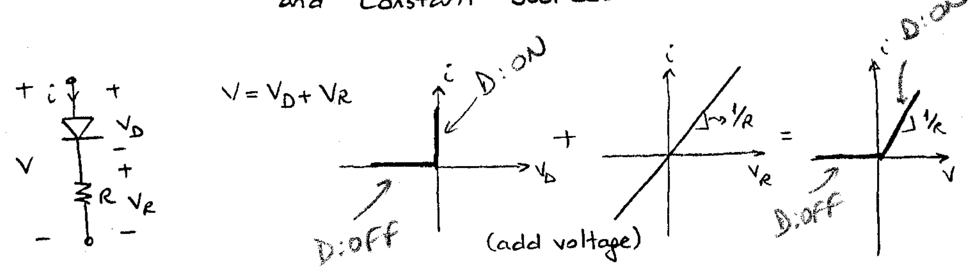
4) Current Controlled Voltage source (CCVS)



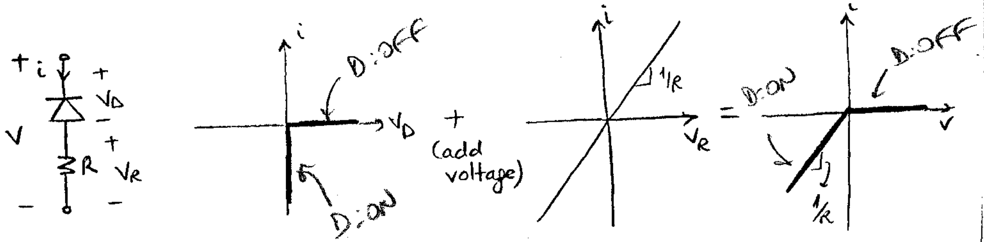
ex $k(i) = 3i^2$

Series and Parallel Connections of Ideal Diodes, Resistors and Constant Sources:

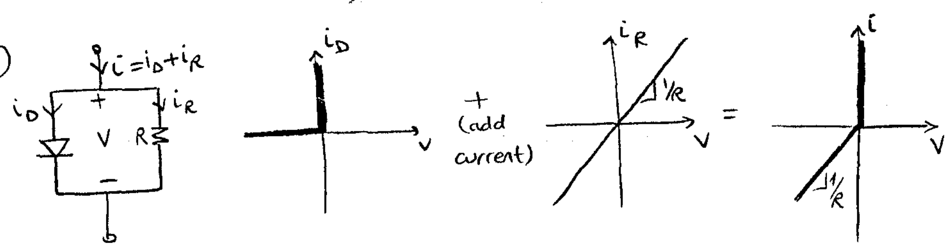
(1)



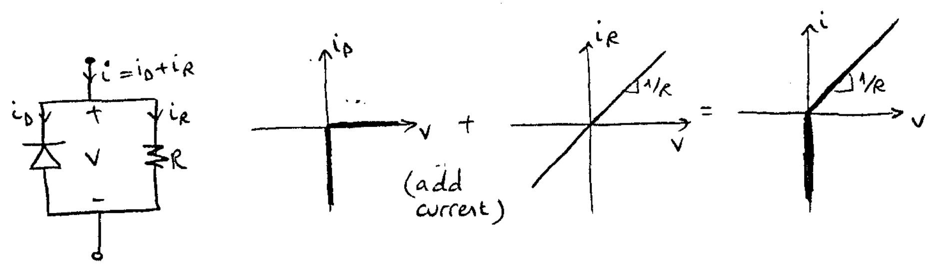
(2)



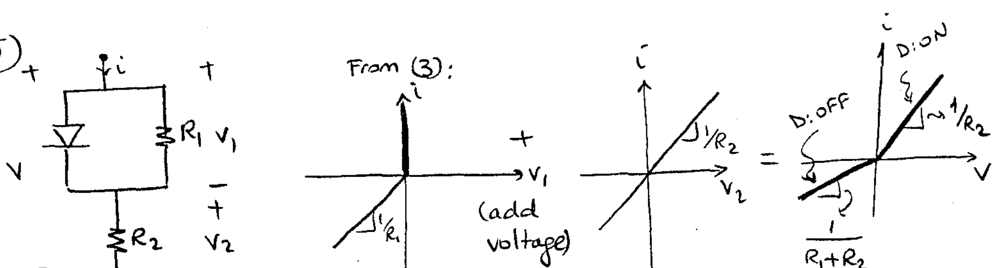
(3)



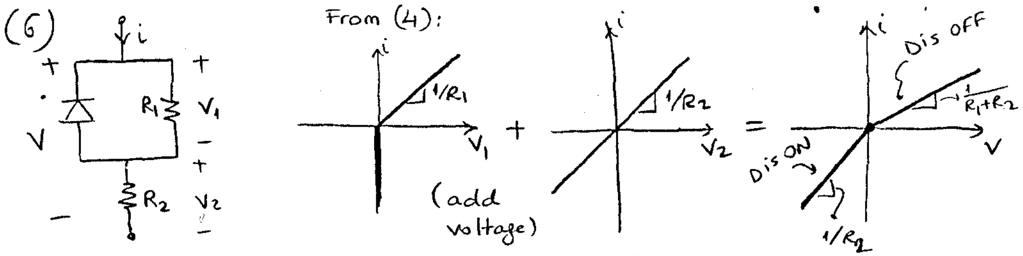
(4)



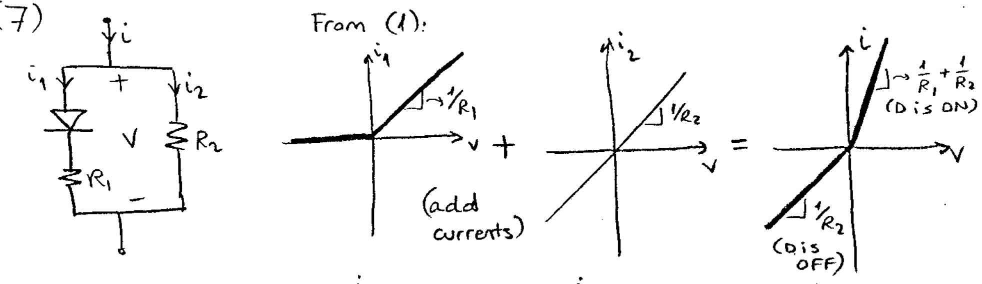
(5)



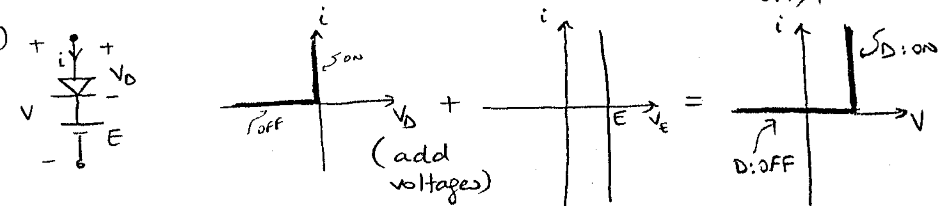
(6)



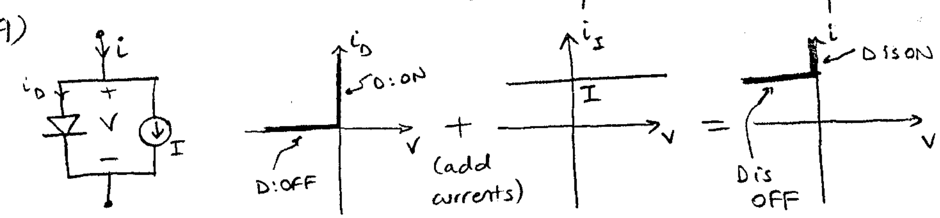
(7)



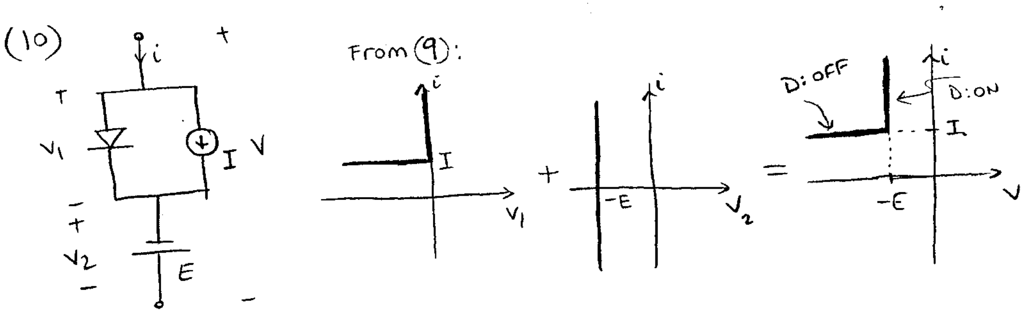
(8)



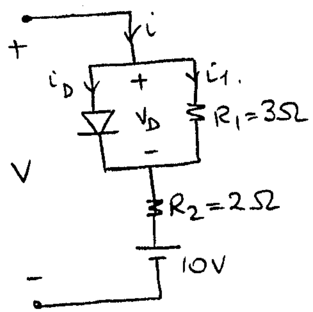
(9)



(10)



Example:



Break Point:

$i_D = 0$ and $V_D = 0$

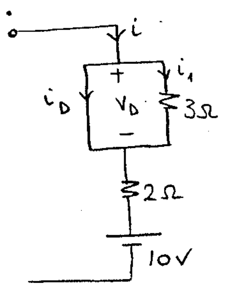
then: $i_1 = 0$ (since $V_D = V_{R_1} = 0$)

$i = 0$

$\Rightarrow V = \underbrace{\cancel{V_D}}_{0} + \underbrace{\cancel{i}}_{0} R_2 + 10 = 10V$

Assume D is ON:

Then $V_D = 0$, $i_D > 0$



$i_1 = \frac{V_D}{3} = 0$

$\Rightarrow i = i_1$

$V = 2i + 10$

$\Rightarrow \boxed{i = \frac{1}{2}V - 5}$

For the diode to be ON:

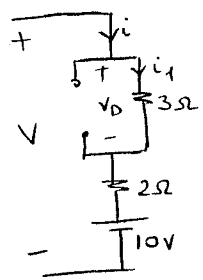
$i_D > 0 \Rightarrow i > 0$

$\Rightarrow \frac{1}{2}V - 5 > 0$

$\Rightarrow \boxed{V > 10}$

Assume D is OFF:

Then $i_D = 0$, $V_D < 0$



$i = i_1$

$V = (3+2)i + 10$

$V = 5i + 10$

$\boxed{i = \frac{1}{5}V - 2}$

$V_D = 3i$

$\Rightarrow V_D = 3 \cdot \left(\frac{1}{5}V - 2\right)$

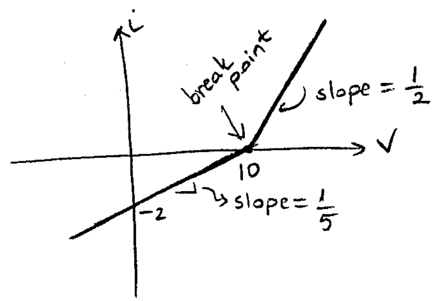
$= \frac{3}{5}V - 6$

For the diode to be OFF:

$V_D < 0 \Rightarrow \boxed{V < 10V}$

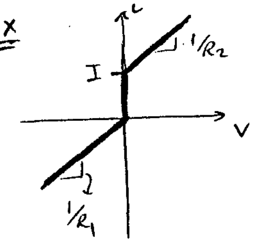
The i-v characteristics is then:

$$i = \begin{cases} \frac{1}{2}V - 5 & , V > 10 \\ 0 & , V = 10 \\ \frac{1}{5}V - 2 & , V < 10 \end{cases}$$



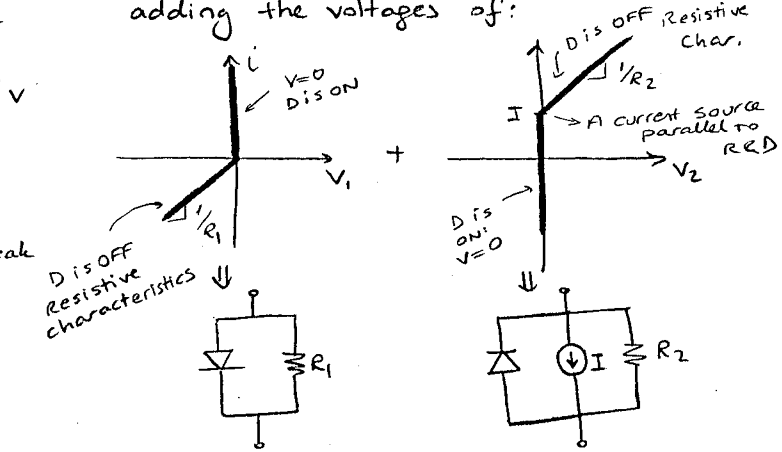
Synthesis (Use Ideal Diodes, Resistors and Constant Sources)

ex

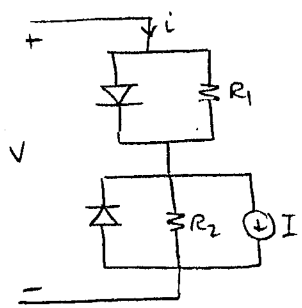


(There are two break points)

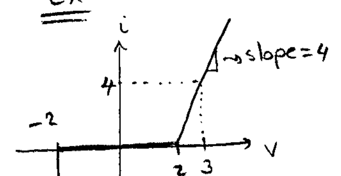
we can obtain this iv characteristics, by adding the voltages of:



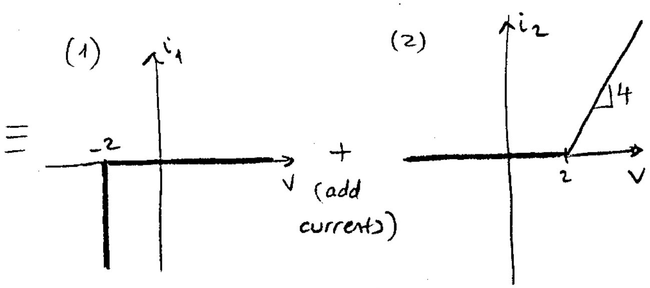
Then:



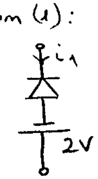
Ex



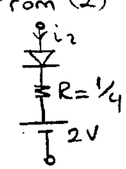
≡



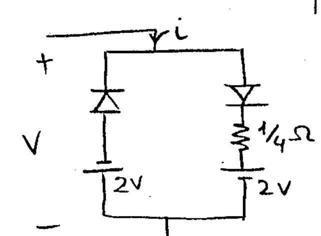
From (1):



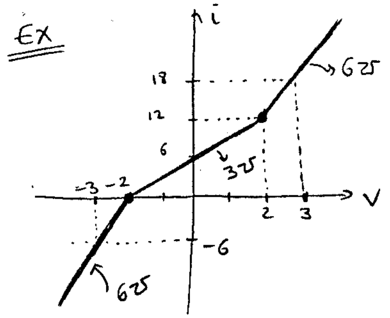
From (2)



⇒

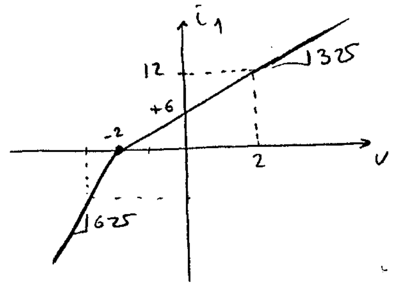


Ex

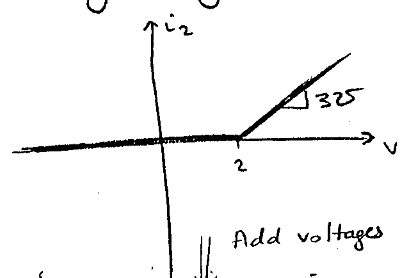


There are two breakpoints, at −2 volts and at 2 Volts.

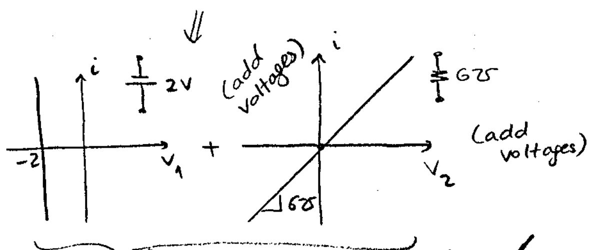
This characteristics can be obtained by adding currents of:

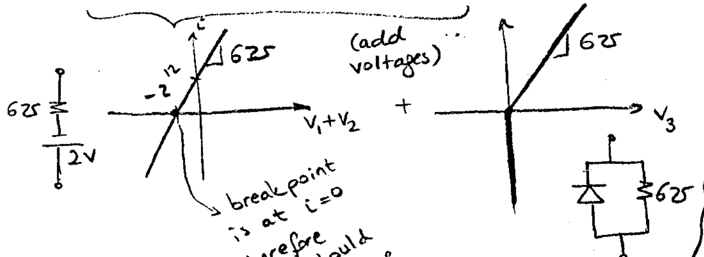


+



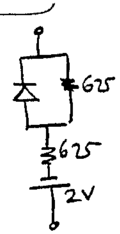
⇓ Add voltages



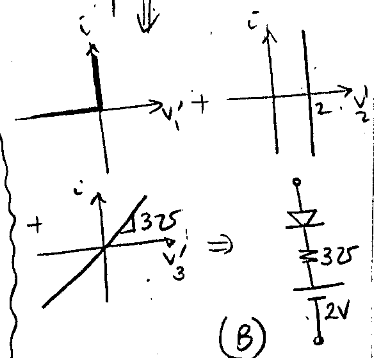


breakpoint is at i=0 therefore slope should change here

Add voltages $(V_1+V_2)+V_3$, you obtain i_1-v characteristics.

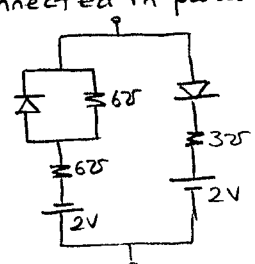


(A)

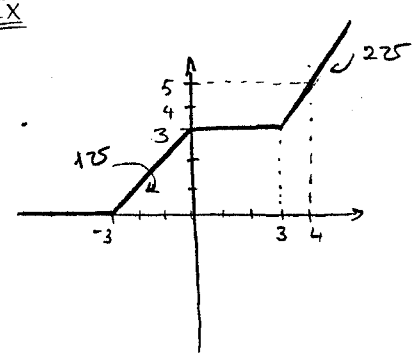


(B)

since we added currents i_1 and i_2 at the beginning, (A) and (B) are connected in parallel:



Ex



Design a circuit made up of ideal diodes, passive resistors and independent sources.

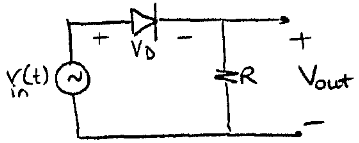
(Hint: there are 3 breakpoints, so you should have 3 diodes in your circuit)

Diode Applications:

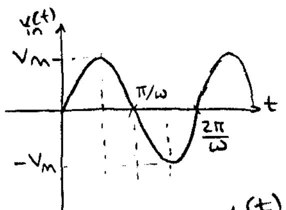
Rectifiers, filters, wave shaping circuits, etc.

Rectifier: The non-linear characteristics of a diode is used to convert alternating current into unidirectional, but pulsating current in the process called rectification.

Half wave rectifier:



$V_{in}(t) = V_m \sin(\omega t)$



1) $0 < t < \frac{\pi}{\omega}$, $v_{in}(t) > 0$

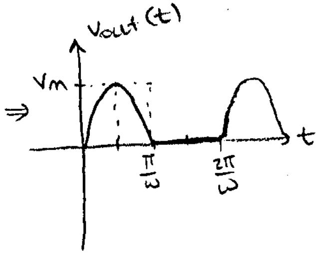
Assume the diode is on:

$V_D = 0$, $i_D = i_R = \frac{V_{in}(t)}{R} > 0$

$V_{out}(t) = V_{in}(t)$

since $i_D > 0$, our claim that D is ON is justified!

$\Rightarrow$



2) $\frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$, $V_{in}(t) < 0$

Assume the diode is OFF:

$i_D = 0 = i_R \Rightarrow V_{out} = 0$

$\Rightarrow V_D = V_{in} - V_{out} < 0$

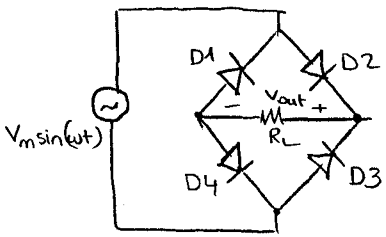
↳ our claim that D is OFF is justified.

$$V_{in,avg} = \frac{1}{T}\int_0^T v_{in}(t)\,dt = 0$$

$$V_{out,avg} = \frac{1}{T}\int_0^{T/2} V_m \sin\omega t\,dt = \frac{V_m}{\pi}$$

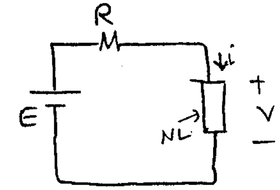
↳ Average (DC) value of the half-wave rectified output voltage.

Exercise:



Find V_{out} and $V_{out,avg}$.

Circuits with a Single Non-Linear Element:



(Typical biasing circuit)

Non-linear element has a characteristic:

I) $i = g(v)$

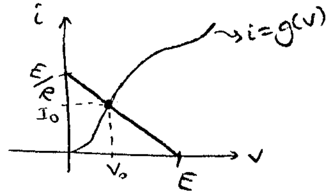
From KVL: $E = Ri + V \Rightarrow \boxed{E = Rg(v) + V}$

II) $v = f(i)$

$\Rightarrow \boxed{E = Ri + f(i)}$

Let's study case I: $E = Rg(v) + V = Ri + V$

Plot the i-v characteristics for R and NL element:



* The intersection point = operating point.

For NL element : $i = g(v)$

For R : $i = \frac{E - V}{R} = -\frac{V}{R} + \frac{E}{R}$

this line is called the "Load line"

Q point : queiescent point.

Load line graphical method is used much in practice to determine operating points of non-linear circuits, because in practice, most of the non-linear circuit problems cannot be solved analytically, and V-i characteristics of a non-linear element is given as a measured curve.

Example: In the above non-linear circuit, $E = 10V$, $R = 100\Omega$ and

$$i = \begin{cases} 0.03v^2 & , \ v \geq 0 \\ 0 & , \ v < 0 \end{cases}$$

Assume $v \geq 0$: $10 = 100(0.03v^2) + v = 0 \Rightarrow 3v^2 + v - 10 = 0$

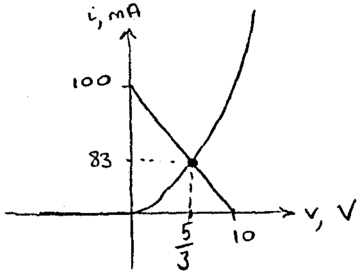
$E = Ri + V$

solving this equation, we find $\rightarrow v = -2V$

$\boxed{v = \frac{5}{3}V > 0}$ solution.

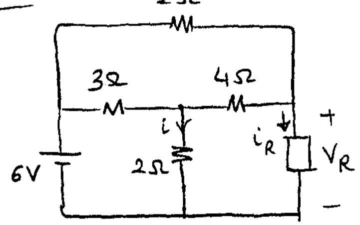
$V_o = \frac{5}{3}V \Rightarrow I_o = 0.03 \cdot \frac{25}{9} \cong 0.083A$

at the operating point, (V_o)



$\frac{E}{R} = \frac{10}{100} = 0.1\ A \Rightarrow 100\ mA$

Example:

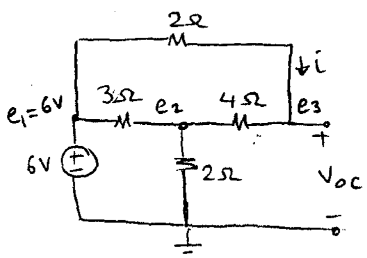


$$i_R = \begin{cases} 0.03 V_R^2 & , V_R \geq 0 \\ 0 & , V_R < 0 \end{cases}$$

Find i.

Solution: Remove the nonlinear element and find the Thevenin equivalent of the rest of the circuit. Then connect R_L and solve for i_R and V_R. Finally, replace the non-linear element with either a voltage source of value V_R, or a current source of value i_R, and solve for i in this new circuit.

Step 1: Find the Thevenin equivalent circuit.



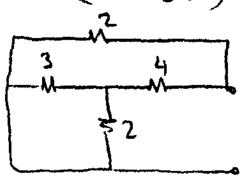
$$\frac{e_2 - 6}{\underset{(2)}{3}} + \frac{e_2}{\underset{(3)}{2}} + \frac{e_2 - 6}{4+2} = 0$$

$$6e_2 = 18 \Rightarrow \boxed{e_2 = 3V}$$

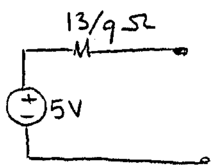
$$i = \frac{6 - e_2}{6} = \frac{1}{2}\ A.$$

$$V_{oc} = V_{th} = e_3 = 6 - 2 * i = \underline{5\ V}$$

$R_{th} = ?$ (kill 6V)

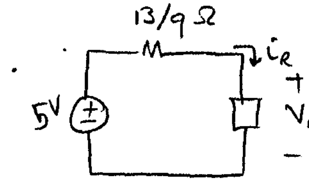


$$R_{th} = 2 \,//\, [4 + (2 // 3)] = \frac{13}{9}\ \Omega$$



= Thevenin Eq.

Step 2: Connect the non-linear load, and find i_R, V_R:



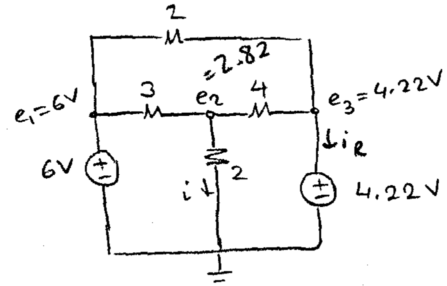
$V \geq 0$: $\quad i_R = 0.03 V_R^2$

$$5 = \frac{13}{9} \cdot (0.03 V_R^2) + V_R$$

$V_R = 4.22V$ ✓ $V_R > 0$ $\qquad$ $V_R = -27.29V$ ✗ $V_R < 0$

$V_R = 4.22\ V$, $\quad i_R = 0.03 * (4.22)^2 = 0.53A$

Step 3: Replace the n.l. load with a voltage (or current) source:



$$\frac{e_2 - 6}{3} + \frac{e_2 - 4.22}{4} + \frac{e_2}{2} = 0$$

$\Rightarrow e_2 = 2.82\ V$

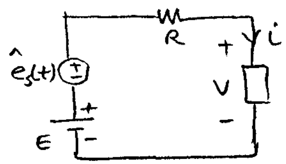
$$i = \frac{e_2}{2} = 1.41\ A$$

Exercise: Find i by replacing the load with a current source of value $i_R = 0.53\ A$.

$$i_R + \underbrace{\frac{4.22 - 2.82}{4}}_{0.35} + \underbrace{\frac{4.22 - 6}{2}}_{-0.89} = 0 \Rightarrow i_R = 0.54\ A!$$

Small Signal Analysis:

A circuit with a DC source and a time-varying input (eg. a sinusoidal waveform) can be analyzed by using small signal analysis method, if the magnitude of input (or sinusoidal signal) is sufficiently small.



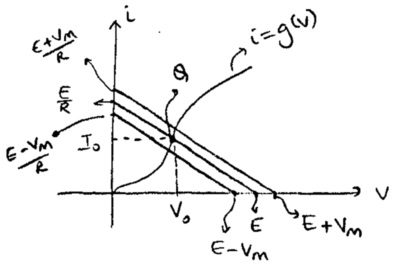
$e_s = E + \hat{e}_s$ where $|\hat{e}_s| << E$

$i = g(v)$

Let $\hat{e}_s(t) = V_m \cos \omega t$.

Then, $-V_m \le \hat{e}_s(t) \le V_m$

and $E - V_m \le e_s \le E + V_m$



* The actual current and voltage vary as a function of time in the neighborhood of the operating point, Q.

$e_s = Ri + v = Rg(v) + v$

If $\hat{e}_s = 0 \Rightarrow e_s = E$ and $E = Rg(V_o) + V_o$, $I_o = g(V_o)$

$v = V_o + \hat{v}(t)$

Expand current in Taylor series about V_o:

$$i = g(v) = g(V_o + \hat{v}) = \underbrace{g(V_o)}_{I_o} + \underbrace{\left.\frac{dg}{dv}\right|_{v=V_o} \hat{v} + \frac{1}{2!} \left.\frac{d^2 g}{dv^2}\right|_{v=V_o} \hat{v}^2 + \dots}_{\hat{i}}$$

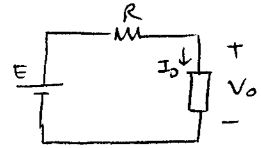
$$e_s = E + \hat{e}_s = R(I_o + \hat{i}) + V_o + \hat{v}$$

$$\hat{i} = \left.\frac{dg}{dv}\right|_{v=V_o} \hat{v} + \underbrace{\text{higher order terms}}_{\text{negligible}}$$

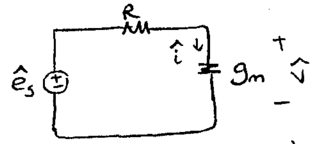
define: $\boxed{g_m \triangleq \left.\frac{dg}{dv}\right|_{v=V_o}}$ → Note that this is the slope of the characteristic of the nonlinear element at the operating point.

$\Rightarrow \boxed{\hat{i} \simeq g_m \hat{v}}$

Now that we used a linear approximation of the non-linear element, we can use superposition.



DC circuit



AC (small-signal) circuit.

Example: In the above circuit:

$E = 10V$, $R = 100\Omega$, $\hat{e}_s = \sin(100t)$

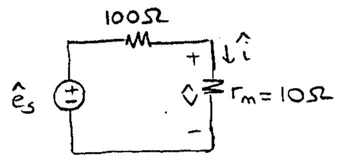
$$i = \begin{cases} 0.03v^2, & v \ge 0 \\ 0 & v < 0 \end{cases}$$

Previously, the DC circuit was solved: $I_o = \frac{250}{3}$ mA, $V_o = \frac{5}{3}$ V

Now, let's solve the AC part:

$$g_m = \left.\frac{di}{dv}\right|_{v=V_o} = \left.\frac{d\,[0.03v^2]}{dv}\right|_{v=5/3} = 0.03 \times 2V_o = 0.1\,\mho = 100\,m\mho$$

$\Rightarrow \boxed{r_m = 10\Omega}$



$\hat{v} = \frac{10}{110} \cdot \hat{e}_s = \frac{1}{11} \sin(100t)$ (V)

$\hat{i} = \frac{\hat{v}}{10} = 9.09 \sin(100t)$ (mA)

$\Rightarrow V = V_o + \hat{v} = \frac{5}{3} + 0.0909 \sin(100t)$ Volts → $V_{max} = 1.757$ V, → $V_{min} = 1.575$ V.

$i = I_o + \hat{i} = \frac{250}{3} + 9.09 \sin(100t)$ (mA) → $I_{max} = 92.42$ mA, → $I_{min} = 74.24$ mA.

To make a comparison, evaluate V when $e_{s,max} = 11$ V and $e_{s,min} = 9$ V.

$e_{s,max} = 11\text{ V} = \overset{R}{100} \cdot 0.03v^2 + V$

$v^2 + \frac{v}{3} - \frac{11}{3} = 0$

$\Rightarrow v = 1.755$ V → V_{max}

$i = 92.445$ mA → I_{max}

$e_{s,min} = 9\text{ V} = 100 \cdot 0.03v^2 + V$

$v^2 + \frac{v}{3} - \frac{9}{3} = 0$

$\Rightarrow v = 1.573$ V → V_{min}

$i = 74.266$ mA → I_{min}